Reimagining Mathematical Modeling Pedagogy: A Systematic Review of Strategies for Preservice Teacher Education

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Abstract

This study delves into the transformative role of mathematical modeling in enhancing mathematics education for preservice teachers. Mathematical modeling, a method that applies mathematical concepts to solve real-world problems, is identified as a pivotal educational tool for bridging the gap between abstract mathematical theories and practical applications. Through a comprehensive qualitative analysis of the literature, this research investigates innovative teaching strategies, including Project-Based Learning (PjBL), Inquiry-Based Learning (IBL), technology-enhanced learning, the flipped classroom model, and interdisciplinary approaches, to ascertain their effectiveness in teaching mathematical modeling. The study underscores educators' challenges, such as mathematical anxiety and resistance to new methodologies, and proposes solutions like professional development and curriculum redesign to overcome these obstacles. By integrating real-world contexts into authentically designed mathematical modeling tasks, the study aims to foster a deeper understanding and appreciation of mathematics among preservice teachers, ultimately preparing them to incorporate mathematical modeling in their future classrooms effectively. The findings suggest that embracing innovative teaching strategies and addressing educational challenges significantly improves the teaching and learning of mathematical modeling, thereby enhancing preservice teachers' mathematical proficiency and problem-solving skills in real-world scenarios.

Introduction

Definition of Mathematical Modeling

Mathematical modeling in mathematics education involves using mathematical concepts, tools, and techniques to represent, analyze, and solve real-world problems. It includes translating real-life scenarios into mathematical formulations, making assumptions, applying mathematical methods to derive solutions, and interpreting the results (Arseven, 2015). This process is crucial for solving mathematical problems and understanding and explaining natural phenomena through a mathematical lens.

The significance of mathematical modeling lies in its ability to bridge the gap between theoretical concepts and practical applications, making mathematics more engaging for learners (Erbaş et al., 2014). It helps students develop critical thinking, problem-solving abilities, and a deeper understanding of mathematical concepts in authentic contexts (Tangkawsakul et al., 2020). Moreover, it fosters interdisciplinary connections by integrating mathematics with other fields such as science, technology, engineering, and social sciences (Kertil & Gürel, 2016).

Mathematical modeling is vital in STEM education by connecting mathematical concepts to real-world problems and applications (Kertil & Gürel, 2016). It encourages creativity, innovation, and collaboration among students, preparing them for careers that demand analytical thinking and problem-solving skills (Tezer, 2020). Through modeling, students learn to make informed decisions, analyze data, and effectively communicate their findings, which are crucial in today's data-driven society.

In summary, mathematical modeling enhances students' mathematical proficiency, problem-solving skills, and real-world application of mathematical concepts, making learning more engaging and relevant across various disciplines.

Importance of Mathematical Modeling in Mathematics Education

Mathematical modeling is valuable for developing students' critical thinking and problem-solving skills and enhancing their ability to apply mathematical concepts in real-world contexts. Engaging in modeling activities presents students with authentic problems that require analysis, interpretation, and solutions using mathematical tools (Kertil & Gürel, 2016). This process encourages critical thinking, informed decision-making, and effective communication of findings (Popović & Lederman, 2015). Students learn to approach problems systematically, break them into manageable parts, and develop logical reasoning skills (Milinković & Ćurčić, 2019).

Modeling fosters creativity and innovation by challenging students to explore multiple solutions and consider different perspectives when addressing real-world issues (Tezer, 2020). It encourages creative thinking, experimentation with various strategies, and adaptation based on the problem's context (Blum, 2015). By engaging in modeling tasks, students gain a deeper understanding of mathematical concepts and their applications in fields like science, technology, and everyday life (Kertil & Gürel, 2016).

Additionally, modeling promotes interdisciplinary connections by integrating mathematics with other subjects, emphasizing its relevance in a broader context. This approach helps students see the interconnectedness of different disciplines and how mathematical concepts apply across various domains (Zhou et al., 2019). Participating in modeling enhances students' mathematical skills and provides a holistic understanding of mathematics' utilization in different professions and industries (Gravemeijer et al., 2017).

In conclusion, mathematical modeling enhances students' mathematical proficiency, critical thinking, and problem-solving abilities while applying concepts in real-world scenarios. Engaging in modeling activities helps students develop essential skills crucial for success in academic, professional, and everyday life contexts.

Rationale for the Study

Mathematical modeling enhances students' understanding of mathematics and fosters problem-solving skills and critical thinking (Arseven, 2015). However, there is a gap in the literature concerning innovative teaching strategies for preservice mathematics teachers. While studies have explored the relationship between mathematics anxiety and preservice teachers' confidence in teaching mathematics and science (Bursal & Paznokas, 2006), research must focus on effective methods to enhance preservice teachers' modeling skills and teaching efficacy.

Preservice mathematics teachers' efficacy beliefs significantly affect their teaching effectiveness (Bates et al., 2011). Studies show that mathematics self-efficacy and teaching efficacy are linked to performance and confidence in teaching mathematics (Bates et al., 2011). Additionally, using technology, pedagogical knowledge, and content knowledge improves preservice secondary mathematics teachers' beliefs and skills in modeling (Kim, 2018). However, there is a lack of research on integrating these factors into innovative teaching strategies for preservice teachers.

The literature also highlights the importance of addressing mathematics anxiety among preservice teachers, as it impacts their teaching efficacy and confidence (Swars et al., 2006). There is a negative relationship between efficacy beliefs about mathematics and mathematics teaching anxiety (Bosica, 2022). Therefore, interventions focusing on reducing anxiety and enhancing self-efficacy could improve preservice teachers' performance in teaching mathematics.

In conclusion, while the role of modeling in mathematics education is well-established, there is a need for research on innovative teaching strategies for preservice teachers. Future studies should focus on developing methods to enhance modeling skills, reduce mathematics anxiety, and improve teaching efficacy.

Research Questions

This study aimed to address the following key research questions to enhance the teaching of mathematical modeling to preservice teachers:

- What are the effective innovative teaching strategies for enhancing preservice teachers' mathematical modeling skills?
- How do these innovative teaching strategies impact preservice teachers' confidence and self-efficacy in teaching mathematical modeling?
- What challenges do educators face in teaching mathematical modeling to preservice teachers, and what solutions can be proposed to overcome these challenges?
- How can real-world contexts be incorporated into mathematical modeling tasks to enhance preservice teachers' learning experiences?

By addressing these research questions, the study seeks to provide a comprehensive understanding of how to effectively teach mathematical modeling to preservice teachers, ultimately preparing them to inspire and educate their future students.

Theoretical Underpinnings

Mathematical modeling in education is linked to foundational educational theories like constructivism, inquiry-based learning, and collaborative learning. These theories collectively enhance the teaching and learning of mathematical concepts by promoting deep and meaningful engagement with mathematics.

Constructivism and Its Influence on Mathematical Modeling

Constructivism posits that learners construct knowledge through experiences and interactions, emphasizing active learning, problem-solving, and real-world application (Supinah & Nuriadin, 2022). This approach's roots trace back to Intuitionist Mathematics, offering insights into understanding and applying mathematical concepts (Kazakçı, 2013). Constructivism fosters mathematical creative thinking abilities (Hidayat et al., 2021) and adapts well to online and digital platforms, enhancing educational practices in critical thinking, problem-solving, and creativity (Charles-Ogan & Wonu, 2019; Juandi et al., 2021; Koptseva, 2020; Zulaika & Syarifuddin, 2018).

Inquiry-Based Learning as a Pedagogical Tool

Inquiry-based learning (IBL) promotes active engagement, exploration, and experimentation within mathematical education (Kogan & Laursen, 2014). This method encourages students to pose questions, investigate problems, and develop solutions through guided inquiry. IBL enhances problem-solving skills and deep conceptual understanding by emphasizing constructing knowledge through personal experiences and exploration (Kwon et al., 2015; Maass et al., 2017; Menezes, 2017; Ramadhani et al., 2021). It also improves cognitive learning outcomes and critical thinking skills (Fitriyah et al., 2020).

The Role of Collaborative Learning Theories

Collaborative learning theories highlight the importance of social interactions and peer collaboration in education (Sofroniou & Poutos, 2016). This approach aligns with constructivist and inquiry-based principles, emphasizing the social construction of knowledge through collaboration (Nelson & Slavit, 2007). Collaborative learning in mathematical modeling enhances understanding and promotes skills such as communication, teamwork, and active participation (Anitha & Kavitha, 2022; Retnowati et al., 2017). It enriches the educational experience, leading to improved problem-solving abilities and a deeper conceptual grasp of mathematics (De Hei et al., 2015; Lu'luilmaknun et al., 2021).

Integrating constructivist, inquiry-based, and collaborative learning theories into mathematical modeling education creates a robust, student-centered framework. This synthesis encourages students to explore, think critically, and solve real-world problems creatively, thereby improving mathematical proficiency and problem-solving skills. By leveraging these theories, educators can design dynamic, interactive learning experiences rooted in practical application, preparing students to address complex mathematical challenges confidently.

Methodology

The methodology for this systematic qualitative review adhered to the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) guidelines, ensuring a comprehensive and transparent approach to reviewing contemporary research literature pertinent to innovative ways of teaching mathematical modeling to preservice teachers of mathematics.

Search Strategy

A meticulous search strategy captured a broad spectrum of relevant literature, encompassing several academic databases and digital libraries renowned for their educational and mathematical research repositories. These included ERIC (Education Resources Information Center), JSTOR, Scopus, and Web of Science. The search was conducted using a combination of keywords and phrases meticulously chosen to encompass the scope of the research question. Keywords such as "mathematical modeling," "preservice mathematics teachers," "innovative teaching methods," and "mathematics education" were used in various combinations to maximize the retrieval of pertinent studies. Boolean operators (AND, OR) were utilized to refine the search, and filters like publication date (from 2000 to present to ensure the currency of the methods discussed) and peer-reviewed status (to ensure academic rigor) were applied.

Selection Criteria

Predefined criteria governed the selection of studies for inclusion in the review to ensure that only literature of the highest relevance and quality was considered. These criteria included:

- *Publication Date:* Only studies published within the last 20 years were considered to ensure the relevance and currency of the teaching methods discussed.
- *Relevance to Research Question:* Studies were selected based on their direct relevance to teaching mathematical modeling to preservice mathematics teachers, explicitly focusing on innovative teaching methods.
- *Academic Rigor:* To ensure the credibility and reliability of the findings, preference was given to peer-reviewed articles published in reputable academic journals.
- *Study Design and Methodology:* Studies that employed robust and appropriate methodologies for investigating teaching methods in mathematics education were included.
- *Geographical Diversity:* Studies conducted in diverse geographical locations were included to capture various educational contexts.

Studies were excluded if they were not published in English, focused solely on in-service teachers without implications for preservice teacher education, or were published in non-peer-reviewed sources.

Data Analysis

The qualitative analysis of the selected studies was performed using the MAXQDA 24 software platform. It involved a synthesis of the findings, methodologies, and theoretical frameworks employed across the body of literature. This synthesis was conducted through a thematic analysis approach, where data were coded and categorized into themes relevant to innovative teaching methods in mathematical modeling for preservice teachers. Themes were identified based on recurring concepts, teaching strategies, and outcomes reported in the studies. This process facilitated the identification of patterns and gaps in the literature, enabling a comprehensive understanding of the current state of research in this area. The synthesis aimed to distill the core principles, pedagogical strategies, and theoretical underpinnings of effective mathematical modeling instruction for preservice teachers, highlighting innovative practices and their implications for teacher education programs.

Findings

The qualitative analysis's findings are presented in Table 1 via main themes, codes, sub-codes, number of references, and actual references consulted. This table provides a comprehensive overview of various pedagogical approaches and strategies for teaching mathematical modeling to preservice teachers based on a review of the academic research literature.

Table 1. The Qualitative Analysis's Findings are Categorized by Main Themes, Codes, Subcodes, Number of References, and Actual References Consulted

Main Theme	Codes	Number of References	Actual References
Traditional Problem-Solving Versus Mathematical Modeling	Traditional Problem Solving	2	Schoenfeld (2016); Erbaş et al. (2016)
	Mathematical Modeling as a Pedagogical Shift	6	Hickendorff (2013); Boero and Dapueto (2007); Dinglasan et al. (2023); Ali et al. (2010); Mokhtari-Hassanabad et al. (2012); Laurens et al. (2017)
Innovative Ways of Teaching Mathematical Modeling	Project-Based Learning	9	Efstratia (2014); Siagian et al. (2019); Septian (2022); Miller (2019); LaForce et al. (2017); Beier et al. (2019); Ratnasari et al. (2018); Prince and Felder (2006); Wahono et al. (2020); Jazuli et al. (2019)
	Inquiry-Based Learning	7	Duran and Dökme (2016); Wu and Lin (2016); Gormally et al. (2009); Lukáč and Sekerák (2016); Maass et al. (2017); Marshall and Horton (2011); Kogan and Laursen (2014)

Main Theme	Codes	Number of References	Actual References
	Technology-Enhanced Learning	9	Natalija et al. (2019); İlhan (2021); Su et al. (2022); Lu and Kaiser (2022); Carrillo-Yañez et al. (2018); Edelen et al. (2020); Erbaş et al. (2014); Kertil and Gürel (2016); Vos (2020)
	Flipped Classroom Approach	6	Li et al. (2017); Lo and Hew (2021); Umam et al. (2019); Lou et al. (2017); Mohd Nasir et al. (2020); Harmini et al. (2022)
	Interdisciplinary Approaches	8	Everingham et al. (2017); Gürbüz and Çalik (2021); Karali (2021); Suh and Han (2019); Vinogradova (2021); Jehlička and Rejsek (2018); Soboleva et al. (2020); Sala Sebastià et al. (2021); Santaolalla et al. (2020)
	Collaborative Learning	5	Retnowati et al. (2017); Anitha and Kavitha (2022); Moreno-Guerrero et al. (2020); Hossein-Mohand et al. (2021); Retnawati et al. (2018)
	Realistic Mathematics Education (RME)	6	Riyanto et al. (2017); Pitriani (2016); Tamur et al. (2020); Setyaningsih et al. (2019); Van Den Heuvel-Panhuizen (2003); Wubbels et al. (1997)
Designing Authentic Mathematical Modeling Tasks	Foundations for Designing Authentic Mathematical Modeling Tasks	12	Giordano et al. (2013); Bliss et al. (2014); NCTM (2018); Lyon and Magana (2020); Post et al. (2004); Heyneman (2003); Tichenor and Sridhar (2019); Rodell et al. (2004); Krahn (2011); Badreddin et al. (2021); Kertil and Gürel (2016); Burkle (2019)
	Incorporating Real-World Contexts into Mathematical Modeling Tasks	5	Popović and Lederman (2015); Kurniadi et al. (2022); Palm (2008); Kabir and Jalali (2021); Voskoglou (2018)
	Authentic Mathematical Modeling Task Examples	16	Boaler (1993); Devlin (2011); NCTM (2014); English and Sriraman (2010); Tsoularis and Wallace (2002); Hethcote (2000); Geneletti (2002); Gomes (2012); Helbing (2001); Ivković et al. (2018); Bauer et al. (2015); Kaveh and Talatahari (2010); Combs and Talegawkar (2013); Blanford et al. (2014); Auerbach and Gorodnichenko (2013); Keller and Fay (2012); Pretty (2008); Grafton and Hussey (2015); Alvarez-Ramirez et al. (2002); Wang et al. (2015)
	Steps Involved in Solving a	4	Haefner (1996); Hartono (2020); Noble (1982); Anderson and Gerbing (1988); Voskoglou (2021); Cheng (2009); Giordano

Main Theme	Codes	Number of References	Actual References
	Mathematical Modeling Problem		et al. (2013)
Identified Challenges and Proposed Solutions	Challenges Faced by Educators	4	Bates et al. (2011); Bursal and Paznokas (2006); Gresham (2018); Baker and Galanti (2017)
	Solutions to Address the Challenges	4	Dunphy (2010); Slavin and Lake (2008); Hopkins et al. (2013); Lewis et al. (2022); Stump (2010); Vail Lowery (2002)

Results

Traditional Problem-Solving Versus Mathematical Modeling

Traditional Problem Solving

Traditional problem-solving methods in mathematics education emphasize algorithmic and procedural approaches. These methods, characterized by a step-by-step process, guide students through predetermined steps to arrive at a solution, emphasizing a structured approach to problem-solving (Schoenfeld, 2016). The influence of George Pólya has been instrumental, advocating for a systematic application of heuristics in solving mathematical problems (Schoenfeld, 2016). Despite their historical significance, these methods have faced criticism for their linear and abstract nature, often detaching from real-life contexts and relying on predetermined formulas and strategies (Erbaş et al., 2016). Consequently, traditional problem-solving approaches may confine learners to pre-established procedures, potentially limiting the development of critical thinking and adaptability in problem-solving scenarios.

Mathematical Modeling as a Pedagogical Shift

Mathematical modeling represents a significant pedagogical evolution, aiming to bridge the gap between abstract mathematical theories and their application in real-world contexts. Students engage in formulating, analyzing, and interpreting mathematical models, applying mathematical concepts to tangible situations, thereby promoting sense-making and metacognition (Hickendorff, 2013). Unlike traditional problem-solving, mathematical modeling encourages students to identify variables, make assumptions, and draw conclusions within real-world scenarios, fostering a cyclic and interdisciplinary process that emphasizes real-world relevance (Boero & Dapueto, 2007; Erbaş et al., 2016). Research shows that incorporating mathematical modeling into education significantly improves students' analytical and problem-solving skills, with comparative studies supporting its superiority over traditional teaching approaches (Ali et al., 2010; Dinglasan et al., 2023; Mokhtari-Hassanabad et al., 2012). Realistic Mathematics Education (RME) embodies the principles of mathematical modeling, prioritizing context and relevance in problem-solving to provide students with meaningful experiences connected to real-world situations, addressing the limitations of abstract mathematics learning (Laurens et al., 2017).

Innovative Ways of Teaching Mathematical Modeling

Project-Based Learning

Project-based learning (PjBL) integrates real-world projects into the curriculum, aligning with constructivist and inquiry-based learning theories to deepen understanding and critical thinking in mathematics education. PjBL engages preservice teachers in projects that require applying mathematical concepts to practical scenarios, enhancing problem-solving abilities, metacognition skills, and mathematical thinking (Efstratia, 2014; Miller, 2019; Septian, 2022; Siagian et al., 2019). Research suggests that PjBL courses positively influence student attitudes and career aspirations in STEM fields (LaForce et al., 2017). Additionally, PjBL promotes the development of essential skills like teamwork, communication, creativity, and the ability to represent mathematical concepts (Beier et al., 2019; Ratnasari et al., 2018). Grounded in educational theories emphasizing active learning and exploration, PjBL empowers preservice teachers to take ownership of their learning, bridging the gap between theoretical knowledge and practical application (Prince & Felder, 2006). The integration of PjBL in STEM education has been particularly effective in improving student learning outcomes, promoting mathematical learning, enhancing problem-solving skills, and understanding spatial concepts (Miller, 2019; Wahono et al., 2020). By incorporating PjBL into mathematics education programs, educators prepare preservice teachers with the knowledge, skills, and confidence to engage students in meaningful mathematical modeling activities (Jazuli et al., 2019).

Inquiry-Based Learning

Inquiry-based learning (IBL) enhances students' critical thinking and problem-solving skills by emphasizing exploration and experimentation (Duran & Dökme, 2016; Wu & Lin, 2016). This student-centered framework allows students to construct meaning actively and communicate mathematical ideas effectively, leading to a deeper comprehension of mathematical concepts. IBL also improves science literacy and confidence, preparing students for complex problem-solving in various contexts (Gormally et al., 2009). Aligning with students' natural curiosity, IBL fosters observation, experimentation, and justification of findings within the mathematics curriculum, enhancing engagement and motivation (Lukáč & Sekerák, 2016). Compared to traditional methodologies, IBL has proven more effective in increasing overall achievement and improving scientific process skills (Maass et al., 2017). Implementing IBL in mathematics education has long-term positive impacts on students' academic trajectories, including improved grades and more strategic course selections (Kogan & Laursen, 2014; Marshall & Horton, 2011). By adopting an IBL approach, educators create a dynamic learning environment that fosters deep engagement with mathematical concepts and cultivates critical thinking and problem-solving skills.

Technology-Enhanced Learning

Technology has transformed the teaching of mathematical modeling by offering innovative approaches to engage students and enhance learning outcomes. Mathematical software, such as GeoGebra, aids in visualizing complex concepts, exploring relationships, and constructing dynamic models, deepening students' understanding of mathematical principles (Natalija et al., 2019). Simulations allow students to conduct experiments and observe outcomes in a controlled environment, making abstract concepts tangible and fostering critical thinking (Natalija et al., 2019). Online platforms provide access to interactive tools and collaborative opportunities, enriching mathematical modeling education (İlhan, 2021; Su et al., 2022). Integrating these tools promotes a contemporary approach to teaching mathematical modeling, enhancing students' understanding and application of mathematical principles (Carrillo-Yañez et al., 2018; Lu & Kaiser, 2022). Quality teaching, supported by empathy and ecological systems theory, highlights the importance of shared realities and emotional connections in the learning process (Edelen et al., 2020). Viewing modeling as both a standalone content area and a vehicle to teach mathematics reflects the diversity in definitions, theoretical backgrounds, and teaching methodologies (Erbaş et al., 2014). Mathematical modeling connects mathematics with other fields like biology and medicine, enhancing problem-solving skills and real-world application (Kertil & Gürel, 2016; Vos, 2020). Integrating digital tools in teaching mathematical modeling offers a multifaceted approach that enhances students' skills and prepares them for a technology-driven world.

Flipped Classroom Approach

The flipped classroom approach inverts traditional teaching methods, emphasizing active and student-centered learning. Students first engage with new material outside of class, typically through instructional videos or readings, with class time dedicated to interactive learning experiences (Li et al., 2017). This approach enhances student engagement and learning outcomes, improving mathematical performance and communication skills (Umam et al., 2019; Lo & Hew, 2021). It also fosters essential academic skills like self-directed learning and problem-solving (Mohd Nasir et al., 2020). The flipped classroom's versatility is evident in its applicability across various mathematical topics and educational levels, suggesting its potential for broad implementation (Harmini et al., 2022). By flipping the traditional classroom model, educators provide a more interactive and personalized learning experience, leading to improved educational outcomes in mathematics education.

Interdisciplinary Approaches

Interdisciplinary approaches in teaching mathematical modeling enhance learning by integrating mathematical concepts with real-world applications from various fields. This strategy increases student engagement, reduces anxiety, and improves achievement (Everingham et al., 2017). By intertwining mathematical modeling with environmental issues, educators highlight its applicability within STEM education, fostering skills for interdisciplinary collaboration and real-world problem-solving (Gürbüz & Çalik, 2021; Kertil & Gürel, 2016). These approaches promote meaningful learning experiences aligned with Education for Sustainable Development (ESD) goals (Karali, 2021; Suh & Han, 2019; Vinogradova, 2021). Leveraging relations between mathematics and other subjects, such as ICT, enhances mathematical thinking through creative projects (Jehlička & Rejsek, 2018; Soboleva et al., 2020). Implementing integrated proposals combining inquiry processes with mathematical modeling helps design effective interdisciplinary teaching sequences, enhancing the educational experience (Sala Sebastià et al., 2021). These methods equip preservice teachers with the skills needed for integrated STEM education, emphasizing the interconnectedness of different subject areas (Santaolalla et al., 2020; Stohlmann et al., 2012). Interdisciplinary approaches prepare students for the complexities of today's world by integrating real-world applications and fostering collaboration across disciplines.

Collaborative Learning

Collaborative learning fosters a collective learning environment where students work together to solve problems, engage in discussions, and build knowledge. This method enhances students' understanding of mathematical concepts through group interaction and collective problem-solving (Retnowati et al., 2017). Collaborative learning improves learning effectiveness and problem-solving abilities, as students working in groups gain a better understanding of mathematical modeling (Anitha & Kavitha, 2022). It also fosters positive attitudes toward mathematics and improves mathematical dimensions (Moreno-Guerrero et al., 2020). Engaging in collaborative learning activities helps students improve essential skills like communication and collaboration, which are critical in mathematical modeling (Hossein-Mohand et al., 2021). Collaborative learning benefits teachers by allowing them to share resources, strategies, and best practices, contributing to their professional development (Retnawati et al., 2018). Integrating collaborative learning into teaching mathematical modeling enhances students' understanding, problem-solving skills, and collaborative competencies, creating a dynamic learning environment.

Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) enhances students' mathematical modeling abilities by starting from real contexts. RME-developed problems are valid and practical for students, significantly improving their modeling abilities (Riyanto et al., 2017; Pitriani, 2016; Tamur et al., 2020). Research shows that child-friendly RME models improve students' activities and elicit positive responses (Setyaningsih et al., 2019). The didactical use of models within RME facilitates understanding of mathematical concepts (Van Den Heuvel-Panhuizen, 2003). Preparing prospective teachers for RME changes their views on mathematics education towards a more inquiry-oriented approach, promoting effective classroom behavior (Wubbels et al., 1997). RME makes mathematics more accessible and enjoyable, improving problem-solving skills and understanding of mathematical concepts.

Designing Authentic Mathematical Modeling Tasks

Foundations for Designing Authentic Mathematical Modeling Tasks

Designing authentic mathematical modeling tasks requires a comprehensive knowledge base from various academic disciplines, including mathematics, economics, environmental science, public health, and engineering. Effective mathematical modeling draws on interdisciplinary principles to address real-world problems (Bliss et al., 2014; Giordano et al., 2013; NCTM, 2018). Educational resources such as textbooks and online materials provide theoretical backgrounds, examples, and exercises for constructing and utilizing models (Giordano et al., 2013; Bliss et al., 2014). Academic research and scientific journals contribute to the development of mathematical models, highlighting the effectiveness of modeling methods in education (Lyon & Magana, 2020). Government and international reports offer data and analysis on global issues, providing models and data for educational tasks (Heyneman, 2003; Krahn, 2011; Post et al., 2004; Rodell et al., 2004; Tichenor & Sridhar, 2019). Knowledge of modeling software tools like MATLAB and Python is crucial for designing realistic tasks (Badreddin et al., 2021). Professional networks and conferences where educators share insights provide contemporary context and inspiration for task design (Kertil & Gürel, 2016). Awareness of global challenges, such as climate change and public health crises, offers rich sources of inspiration for creating relevant modeling tasks (Burkle, 2019).

Incorporating Real-World Contexts into Mathematical Modeling Tasks

Incorporating real-world contexts into mathematical modeling tasks significantly enhances the teaching and learning of mathematical concepts. This approach bridges the gap between abstract theories and practical applications, providing students with a more comprehensive and engaging learning experience. Introducing real-world data and scenarios makes abstract concepts tangible and relatable, motivating students and fostering a deeper understanding (Kurniadi et al., 2022; Popović & Lederman, 2015). Applying mathematical tools to real-world problems improves problem-solving skills and prepares students for diverse scenarios, encouraging critical and creative thinking (Kurniadi et al., 2022; Popović & Lederman, 2015). Authentic tasks enhance student engagement, as they are more likely to be invested in outcomes when considering real-world factors (Palm, 2008). Linking theory to practice by incorporating real-life situations makes learning more relevant and demonstrates the dynamic utility of mathematical modeling (Kabir & Jalali, 2021; Voskoglou, 2018). This educational strategy equips students with the skills necessary for future success, leveraging interdisciplinary knowledge, educational resources, and case studies to create meaningful learning experiences (Kabir & Jalali, 2021; Voskoglou, 2018).

Authentic Mathematical Modeling Task Examples

The mathematical modeling tasks presented are designed with precision, integrating the principles of applying mathematical concepts to real-world scenarios, as emphasized by Boaler (1993). These tasks are academic exercises and critical tools for engaging with and addressing global challenges, underscoring the importance of mathematical concepts and modeling techniques in fostering critical thinking and problem-solving skills (Devlin, 2011; NCTM, 2014). Furthermore, the interdisciplinary nature of these tasks enriches the educational experience by weaving together knowledge from mathematics, science, economics, and technology, offering a comprehensive approach to learning (English & Sriraman, 2010). Each task is outlined with a clear objective, data, and procedural steps, ensuring teacher candidates apply these concepts effectively to practical problems.

Task 1: Population Growth Modeling (Tsoularis & Wallace, 2002)

- *Objective:* Model the growth of a local wildlife population over the next decade to understand the impact of conservation efforts.

- *Data:* Historical population data for the past 20 years, birth and death rates, and data on factors affecting these rates (e.g., predation, disease, habitat changes).
- *Task Steps:*
    - Analyze the historical population data to identify trends.
    - Apply a logistic growth model to predict future population sizes, considering current conservation efforts.
    - Discuss the implications of the model's predictions for future conservation strategies.

Task 2: Spread of Infectious Diseases (Hethcote, 2000)

- *Objective:* Model the spread of a new infectious disease in a community to determine the effectiveness of different intervention strategies.
- *Data:* Initial number of infected individuals, transmission rate, recovery rate, and vaccination rate.
- *Task Steps:*
    - Use the SIR model to simulate the spread of the disease over time.
    - Modify the model to include intervention strategies, such as increased vaccination rates or social distancing measures.
    - Analyze how these strategies impact the number of infected individuals over time.

Task 3: Environmental Impact Assessment (Geneletti, 2002)

- *Objective:* Model the impact of plastic waste on ocean health over the next 50 years and evaluate the effectiveness of different reduction strategies.
- *Data:* Current levels of plastic pollution, rates of plastic degradation, impact on marine life, and data on plastic consumption and waste management practices.
- *Task Steps:*
    - Develop a model to predict future levels of plastic pollution in the ocean.
    - Simulate the effects of various waste reduction and recycling strategies on these levels.
    - Assess the long-term impacts of these strategies on marine ecosystems.

Task 4: Financial Modeling for Personal Savings (Gomes, 2012)

- *Objective:* Model the growth of personal savings over 20 years based on different investment strategies.
- *Data:* Initial savings amount, average annual return rates for different investment options (e.g., stocks, bonds, savings accounts), and annual contribution to savings.
- *Task Steps:*
    - Calculate the future value of the savings for each investment option using compound interest formulas.
    - Compare the results to determine which investment strategy yields the highest return.
    - Discuss the risk factors associated with each investment option.

Task 5: Traffic Flow Analysis for a City Intersection (Helbing, 2001)

- *Objective:* Model traffic flow through a busy city intersection to propose improvements that could reduce congestion.
- *Data:* Traffic count data for different times of day, average vehicle speed, and intersection layout.
- *Task Steps:*
    - Use graph theory to model the traffic flow through the intersection.
    - Simulate the effect of changes to the intersection, such as adding a traffic light or creating a roundabout.
    - Evaluate which changes most effectively improve traffic flow and reduce congestion.

Task 6: Sports Performance Analysis in Basketball (Ivković et al., 2018)

- *Objective:* Analyze shooting performance data to model the optimal shooting angles and positions on the basketball court.
- *Data:* Player shooting data (shot position, angle, and success rate), player height, and arm reach.
- *Task Steps:*
    - Collect and organize shooting data by player position and shot success rate.
    - Use trigonometry and physics to model the basketball trajectory for different shooting angles and positions.
    - Determine the optimal shooting angles and positions for maximizing shot success rate.

Task 7: Weather Forecasting Model (Bauer et al., 2015)

- *Objective:* Develop a model to predict local weather conditions (temperature, precipitation) based on historical weather data for the next week.
- *Data:* Historical weather data (temperature, humidity, pressure, wind speed) for the past decade.
- *Task Steps:*
    - Analyze historical data to identify patterns and correlations between different weather variables.
    - Apply regression analysis or time series forecasting methods to predict future weather conditions.
    - Evaluate the model's accuracy by comparing its predictions with actual weather conditions.

Task 8: Bridge Design Optimization (Kaveh & Talatahari, 2010)

- *Objective:* Use mathematical modeling to design a bridge that optimizes material use while ensuring structural integrity and safety.
- *Data:* Material strength, load capacity requirements, environmental factors (wind, water flow), and budget constraints.
- *Task Steps:*
    - Model the forces acting on the bridge, including tension, compression, and shear forces, using principles of physics and engineering.
    - Optimize the design by adjusting dimensions and materials to meet safety standards while minimizing costs.

- Analyze the impact of environmental factors on the design and incorporate necessary adjustments.

Task 9: Optimization of School Lunch Nutrition (Combs & Talegawkar, 2013)

- *Objective:* Model an optimal school lunch menu that meets nutritional guidelines at minimal cost.
- *Data:* Nutritional content and cost of various food items, dietary guidelines, and student preferences.
- *Task Steps:*
  - Define nutritional targets based on dietary guidelines.
  - Use linear programming to create a menu that meets nutritional targets at the lowest possible cost, considering student preferences.
  - Evaluate the solution's feasibility and adjust the model to incorporate additional constraints, such as variety or food allergies.

Task 10: Modeling the Impact of Renewable Energy Adoption (Blanford et al., 2014)

- *Objective:* Model the impact of increasing renewable energy sources on the reduction of carbon emissions over the next 20 years.
- *Data:* Current energy consumption data, carbon emission rates for different energy sources, and potential growth rates for renewable energy adoption.
- *Task Steps:*
  - Analyze current energy consumption patterns and associated carbon emissions.
  - Model the transition to renewable energy sources, considering various adoption rates and technological advancements.
  - Calculate the projected reduction in carbon emissions and discuss the implications for environmental policy and sustainable development.

Task 11: Economic Impact of a New Policy (Auerbach & Gorodnichenko, 2013)

- *Objective:* Model the potential economic impact of a new tax policy on a local economy over the next five years.
- *Data:* Current economic indicators (GDP, unemployment rate, inflation rate), historical data on similar policies, and consumer spending habits.
- *Task Steps:*
  - Use historical data to establish a relationship between tax policy changes and economic indicators.
  - Apply econometric models to simulate the new policy's impact on GDP, unemployment, and inflation.
  - Assess the policy's potential effects on consumer spending and overall economic health.

Task 12: Analysis of Social Media Influence (Keller & Fay, 2012)

- *Objective:* Model the spread of information on social media platforms and its impact on public opinion regarding a specific topic.

- *Data:* Social media post frequencies, engagement rates (likes, shares, comments), and sentiment analysis of comments.
- *Task Steps:*
    - Construct a network model representing users and their interactions.
    - Use diffusion models to simulate the spread of information through the network.
    - Analyze how changes in engagement rates affect the spread of information and public opinion.

Task 13: Sustainable Farming Practices (Pretty, 2008)

- *Objective:* Model the long-term effects of various farming practices on soil health and crop yields.
- *Data:* Soil quality indicators, crop yield records, and data on farming practices (crop rotation, use of fertilizers, etc.).
- *Task Steps:*
    - Develop models to represent the relationship between farming practices and soil quality over time.
    - Simulate the impact of sustainable practices on soil health and crop yields.
    - Compare the long-term sustainability and economic viability of different farming practices.

Task 14: Water Resource Management (Grafton & Hussey, 2015)

- *Objective:* Develop a model to optimize the allocation of water resources in a drought-prone region to meet agricultural, industrial, and residential needs.
- *Data:* Water usage, rainfall records, population growth trends, and agricultural water needs.
- *Task Steps:*
    - Create a demand-supply model for water resources considering various sectors.
    - Apply optimization techniques to allocate water resources efficiently under scarcity conditions.
    - Evaluate the model's effectiveness in ensuring water availability and sustainability.

Task 15: Modeling the Dynamics of a Stock Market (Alvarez-Ramirez et al., 2002)

- *Objective:* Develop a model to predict stock market trends based on historical data and current economic indicators.
- *Data:* Historical stock prices, trading volumes, economic indicators (interest rates, inflation rates), and news sentiment analysis.
- *Task Steps:*
    - Analyze historical data to identify patterns and correlations with economic indicators.
    - Use machine learning techniques to develop a predictive model for stock market trends.
    - Assess the model's accuracy by back testing with historical data and refine the model based on performance.

Task 16: Urban Traffic Congestion Mitigation (Wang et al., 2015)

- *Objective:* Model the impact of various congestion mitigation strategies on urban traffic flow and air quality.

- *Data:* Traffic density data, vehicle emission rates, public transportation usage statistics, and city layout.
- *Task Steps:*
    - Simulate traffic flow using fluid dynamics models and city layout data.
    - Evaluate the effectiveness of congestion mitigation strategies (e.g., congestion pricing, improved public transportation) on traffic flow and air quality.
    - Recommend policy measures based on the simulation results to reduce traffic congestion and improve air quality.

Steps Involved in Solving a Mathematical Modeling Problem

Solving a mathematical modeling problem involves transforming complex real-world scenarios into mathematical frameworks to derive interpretable and applicable solutions. The process consists of several steps, each supported by relevant scholarly work:

- *Understanding the Problem:* The initial focus is on grasping the essence of the problem by defining the model's goal, pinpointing crucial questions, and identifying pertinent variables and constraints. Haefner (1996) emphasizes the importance of this foundational step for successful modeling (Haefner, 1996).
- *Formulating the Model:* Once the problem is understood, a mathematical model is developed to represent the scenario mathematically. This involves selecting suitable mathematical tools and techniques based on the identified objectives and constraints. Hartono (2020) highlights the significance of applying theoretical constructs to practical scenarios during this step (Hartono, 2020).
- *Analyzing the Model:* The next step involves analyzing the model to understand its behavior and properties using mathematical reasoning, simulations, or computational methods to explore solutions or predictions. Noble (1982) underscores the role of this phase in linking mathematical models with creative problem-solving approaches (Noble, 1982).
- *Interpreting the Results:* The mathematical outcomes must be translated back into the context of the original problem, evaluating their practical implications or predictions. Anderson and Gerbing (1988) advocate for critically assessing the results for their applicability and relevance (Anderson & Gerbing, 1988).
- *Validating the Model:* To ensure accuracy, the model's predictions are compared against real-world data or outcomes, often involving statistical analysis or additional experimentation. Voskoglou (2021) emphasizes systematic validation to confirm the model's reliability and applicability (Voskoglou, 2021).
- *Refining the Model:* Based on validation insights, the model is refined and adjusted as necessary. This iterative improvement process enhances the model's accuracy and relevance. Cheng (2009) discusses the need for incorporating new data and feedback to bridge theoretical models with real-life problem-solving (Cheng, 2009).
- *Implementing the Solution:* The final step involves applying the validated model to address the original problem or inform decision-making based on its predictions. Giordano et al. (2013) emphasize the practical application of modeling insights in real-world scenarios, underscoring the role of

mathematical modeling across various fields (Giordano et al., 2013).

These steps collectively highlight the iterative and interdisciplinary nature of mathematical modeling. The process requires a blend of mathematical theory, computational skills, and domain-specific knowledge, illustrating its capacity to foster creativity, ensure meticulous planning, and facilitate comprehensive problem-solving strategies.

Identified Challenges and Proposed Solutions

Challenges Faced by Educators

Educators face several obstacles when teaching mathematical modeling to preservice teachers. One significant issue is mathematical anxiety and self-efficacy. Preservice teachers often experience math anxiety, which affects their confidence and performance in teaching mathematics, ultimately impacting the quality of their instruction (Bates et al., 2011; Bursal & Paznokas, 2006).

Additionally, resistance to new teaching methods is common among preservice teachers, often exacerbated by anxiety and self-efficacy issues (Gresham, 2018). Many also struggle with understanding complex mathematical concepts involved in modeling, such as fractions, decimals, and percentages, requiring targeted support and instructional strategies (Baker & Galanti, 2017). Another significant barrier is the lack of adequate resources, technology, and manipulatives to effectively teach mathematical modeling, hindering the implementation of hands-on activities and real-world applications crucial for effective learning (Bursal & Paznokas, 2006).

Solutions to Address the Challenges

To overcome these challenges, research supports several solutions. Engaging educators in targeted professional development programs enhances their pedagogical skills, content knowledge, and confidence. These programs should emphasize inquiry-based learning, collaborative teaching practices, and effective use of technology (Dunphy, 2010; Slavin & Lake, 2008). Redesigning the curriculum to incorporate interdisciplinary approaches, hands-on activities, and real-world mathematical modeling applications significantly improves preservice teachers' learning experiences. Aligning the curriculum with STEM education best practices and integrating mathematical modeling across different subjects are recommended strategies (Hopkins et al., 2013).

Collaboration among educators, mathematics coaches, and professional learning communities creates a supportive environment, enabling the sharing of best practices, resources, and instructional strategies, helping to overcome limited resources and resistance to new teaching methods (Lewis et al., 2022; Stump, 2010). Integrating interdisciplinary approaches into teaching helps preservice teachers make cross-curricular connections and deepens their understanding of mathematical concepts. Employing real-world applications and hands-on activities across multiple disciplines offers a holistic view of mathematical modeling (Vail Lowery, 2002).

Discussion

Traditional Problem-Solving Versus Mathematical Modeling

Traditional problem-solving in mathematics education emphasizes algorithmic and procedural approaches, guiding students through step-by-step processes to arrive at solutions (Schoenfeld, 2016). George Pólya's influence has been significant, advocating for systematic heuristics in problem-solving (Schoenfeld, 2016). However, these methods often face criticism for their linear and abstract nature, which can detach students from real-life contexts and confine them to pre-established procedures, potentially limiting critical thinking and adaptability (Erbaş et al., 2016). In contrast, mathematical modeling represents a pedagogical shift that bridges abstract mathematical theories with real-world applications. It encourages students to identify variables, make assumptions, and draw conclusions within real-world scenarios, promoting sense-making and metacognition (Hickendorff, 2013). Research shows that incorporating mathematical modeling enhances analytical and problem-solving skills more effectively than traditional methods (Ali et al., 2010; Dinglasan et al., 2023; Mokhtari-Hassanabad et al., 2012). Realistic Mathematics Education (RME) embodies these principles by providing meaningful experiences connected to real-world situations, addressing the limitations of abstract learning (Laurens et al., 2017).

Innovative Ways of Teaching Mathematical Modeling

Innovative teaching methods such as Project-Based Learning (PjBL), Inquiry-Based Learning (IBL), technology-enhanced learning, the flipped classroom approach, interdisciplinary approaches, and collaborative learning significantly enhance the teaching of mathematical modeling. PjBL integrates real-world projects into the curriculum, promoting active engagement and practical application of mathematical concepts, thereby improving problem-solving skills and metacognition (Efstratia, 2014; Miller, 2019; Siagian et al., 2019). IBL emphasizes exploration and experimentation, enhancing critical thinking and scientific literacy (Duran & Dökme, 2016; Wu & Lin, 2016). Technology-enhanced learning tools, such as GeoGebra and simulations, provide interactive and dynamic ways to visualize and apply mathematical principles (Natalija et al., 2019). The flipped classroom model fosters active learning by dedicating class time to interactive activities, improving student engagement and performance (Li et al., 2017; Umam et al., 2019). Interdisciplinary approaches integrate mathematical concepts with real-world applications from various fields, enhancing student engagement and achievement (Everingham et al., 2017). Collaborative learning environments promote group interaction and collective problem-solving, improving understanding and essential skills like communication and collaboration (Retnowati et al., 2017; Anitha & Kavitha, 2022). These innovative methods create dynamic and engaging learning experiences, preparing students for real-world applications.

Designing Authentic Mathematical Modeling Tasks

Designing authentic mathematical modeling tasks involves a comprehensive knowledge base from various disciplines such as mathematics, economics, environmental science, public health, and engineering. Effective modeling draws on interdisciplinary principles to address real-world problems, supported by educational resources, academic research, government reports, and technology tools (Bliss et al., 2014; Giordano et al., 2013; NCTM, 2018; Lyon & Magana, 2020). Incorporating real-world contexts into these tasks bridges the gap between abstract theories and practical applications, enhancing understanding and engagement (Kurniadi et al., 2022; Popović & Lederman, 2015). This approach improves problem-solving skills by applying mathematical tools to real-world problems, fostering critical and creative thinking (Palm, 2008). Linking theory to practice makes learning more relevant and demonstrates the utility of mathematical modeling, equipping students with essential skills for future success (Kabir & Jalali, 2021; Voskoglou, 2018). Real-world data and scenarios motivate students and make abstract concepts tangible and relatable, creating meaningful and applicable learning experiences.

Identified Challenges and Proposed Solutions

Educators face several challenges in teaching mathematical modeling to preservice teachers, including mathematical anxiety, resistance to new teaching methods, conceptual difficulties, and limited resources (Bates et al., 2011; Bursal & Paznokas, 2006; Gresham, 2018; Baker & Galanti, 2017). Addressing these challenges requires targeted solutions. Professional development programs can enhance educators' pedagogical skills and confidence, emphasizing inquiry-based learning and effective use of technology (Dunphy, 2010; Slavin & Lake, 2008). Curriculum redesigns that incorporate interdisciplinary approaches, hands-on activities, and real-world applications can improve learning experiences (Hopkins et al., 2013). Collaborative learning experiences among educators foster a supportive environment for sharing best practices and resources (Lewis et al., 2022; Stump, 2010). Integrating interdisciplinary approaches helps preservice teachers make cross-curricular connections and deepen their understanding of mathematical concepts, employing real-world applications and hands-on activities to offer a holistic view of mathematical modeling (Vail Lowery, 2002). These strategies collectively enhance the quality of mathematical modeling education and prepare educators to effectively teach and engage their students.

Suggestions for Practice

To enhance the teaching and learning of mathematical modeling in preservice teacher education programs, the following recommendations are proposed:

- Incorporate Project-Based Learning (PjBL) into the Curriculum
    - It is essential to include authentic projects requiring the application of mathematical concepts to solve real-world problems.
    - Facilitating interdisciplinary projects that integrate knowledge from various subjects is crucial for enhancing problem-solving and analytical skills.
- Adopt Inquiry-Based Learning Approaches
    - Encouraging preservice teachers to pose questions, explore, and experiment as part of the learning process fosters deep engagement with mathematical concepts.
    - Providing opportunities for students to lead investigations and present their findings promotes a sense of ownership and active participation.

- Leverage Technology-Enhanced Learning Tools
    - Using digital tools, such as simulations, mathematical software (e.g., GeoGebra), and online collaborative platforms, helps visualize and explore mathematical concepts.
    - Facilitating remote and blended learning opportunities through technology ensures access to diverse resources and mathematical modeling activities.
- Implement the Flipped Classroom Model
    - Preparing instructional videos or readings for preservice teachers to study outside of class allows in-class time for hands-on activities, discussions, and application of concepts.
    - Promoting collaborative problem-solving and critical thinking during class sessions utilizes the group's collective knowledge to explore complex scenarios.
- Utilize Interdisciplinary Approaches
    - Connecting mathematical modeling with subjects such as science, technology, engineering, and social sciences illustrates the multifaceted application of mathematics.
    - Collaborating with educators from other disciplines creates cohesive and integrated learning experiences that highlight the relevance of mathematics in addressing societal challenges.
- Promote Collaborative Learning Environments
    - Organizing preservice teachers into small groups for collaborative projects and problem-solving activities emphasizes the importance of communication and teamwork.
    - Fostering a classroom culture that values diverse perspectives and collective knowledge-building encourages students to learn from each other.
- Engage with Realistic Mathematics Education (RME) Principles
    - Starting with real-life contexts and problems relatable for preservice teachers forms the basis for developing mathematical modeling tasks.
    - Focusing on the didactical use of models and tools facilitates a deeper understanding of mathematical concepts, guiding preservice teachers in applying these approaches in their future classrooms.
- Design Authentic Mathematical Modeling Tasks
    - Developing tasks that incorporate real-world data and scenarios challenges preservice teachers to apply mathematical concepts in practical, meaningful contexts.
    - Encouraging the exploration of current global issues through mathematical modeling enhances the relevance and urgency of mathematics education.
- Address Challenges Proactively
    - Offering targeted professional development opportunities for educators enhances their understanding and teaching of mathematical modeling.
    - Creating a supportive learning environment that addresses mathematical anxiety encourages experimentation and risk-taking in problem-solving.
- Foster Continuous Reflection and Adaptation
    - Encouraging preservice teachers and educators to reflect on their learning and teaching experiences helps identify areas for improvement and innovation.
    - Adapting teaching strategies and curriculum content based on feedback and evolving

educational research continuously enhances the effectiveness of mathematical modeling instruction.

Implementing these suggestions will significantly advance mathematics education, preparing preservice teachers to teach mathematical modeling effectively and fostering a generation of students proficient in applying mathematics to solve complex real-world problems.

Recommendations for Teacher Education Programs

In light of the key findings presented in this study, several recommendations are proposed to enhance teacher education programs, particularly in the context of mathematical modeling:

- Integrate Mathematical Modeling into the Curriculum
    - It is essential to embed mathematical modeling tasks and projects throughout the teacher education curriculum to ensure preservice teachers gain hands-on experience applying mathematical concepts to real-world problems.
    - Emphasizing the importance of mathematical modeling as a critical competency in modern mathematics education highlights its relevance across various disciplines.
- Promote Active Learning Strategies
    - Adopting active learning strategies such as Project-Based Learning (PjBL) and Inquiry-Based Learning (IBL) fosters a deeper understanding of mathematical concepts and improves problem-solving skills.
    - Providing opportunities for preservice teachers to design and implement their own mathematical modeling projects allows for reflection on their learning process and outcomes.
- Leverage Technology-Enhanced Learning
    - It is important to incorporate technology-enhanced learning tools and resources, including mathematical software and online platforms, to facilitate the teaching and learning of mathematical modeling.
    - Offering training sessions helps develop preservice teachers’ competencies in using digital tools effectively within the mathematics classroom.
- Implement the Flipped Classroom Approach
    - Encouraging the use of the flipped classroom model maximizes classroom time for active, student-centered learning experiences.
    - Preparing preservice teachers to create instructional videos and develop activities reinforces mathematical concepts through application and discussion.
- Foster Interdisciplinary Approaches
    - Integrating interdisciplinary approaches within the teacher education program demonstrates how mathematical modeling applies to real-world problems across various fields.
    - Collaborating with other departments helps design cross-curricular projects that highlight the interconnectedness of mathematics with science, technology, engineering, and beyond.
- Encourage Collaborative Learning

    - Creating a collaborative learning environment where preservice teachers work in teams to solve mathematical modeling tasks facilitates the development of communication and teamwork skills.
    - Introducing collaborative projects that require engagement with peers, educators, and professionals from other disciplines enriches the learning experience.
- Adopt Realistic Mathematics Education (RME) Principles
    - Training preservice teachers in the principles of RME focuses on using real contexts as a starting point for learning and developing mathematical modeling abilities.
    - Encouraging the design and implementation of RME tasks ensures they are both valid and practical for students.
- Provide Professional Development Opportunities
    - Offering continuous professional development opportunities focuses on the latest pedagogical strategies, technological tools, and research in mathematical modeling.
    - Facilitating participation in workshops, seminars, and conferences allows preservice teachers to stay current with educational trends and network with mathematics educators.
- Emphasize Reflective Practice
    - Encouraging reflective practice requires preservice teachers to critically assess their mathematical modeling projects and teaching strategies.
    - Implementing a portfolio or journaling system where preservice teachers document their experiences, challenges, successes, and areas for growth enhances their professional development.

By implementing these recommendations, teacher education programs will significantly enhance the preparation of preservice teachers, equipping them with the necessary tools and approaches to teach mathematical modeling effectively. This holistic preparation will empower future educators to inspire their students with a deep understanding and appreciation of mathematics and its real-world applications.

Recommendations for Future Research

The literature on mathematics education, particularly concerning teaching mathematical modeling to preservice teachers, identifies several critical areas for future research. Exploring these areas is crucial for advancing pedagogical strategies, integrating technology in education, and developing teacher competencies and efficacy. The suggested directions for future research include:

- Long-Term Impacts of Innovative Teaching Strategies
    - Investigating the enduring effects of innovative teaching methods on preservice teachers' competencies in mathematical modeling is essential.
    - Understanding the long-term influence of these strategies on teaching efficacy, mathematical performance, and confidence is crucial for refining teacher education programs.
- Effectiveness of Virtual Fieldwork
    - Exploring the potential and limitations of virtual fieldwork, especially under conditions that

limit traditional fieldwork, is important.
    - Research in this area guides technology integration into teacher education programs and enhances preservice teachers' learning experiences.
- Role of Mastery Experiences in Teaching Efficacy
    - Examining the impact of mastery experiences, particularly cognitive mastery, on the teaching efficacy of preservice teachers for mathematics and science is vital.
    - Insights from this research inform targeted interventions and professional development programs to improve teaching efficacy.
- Development of Digital Competencies through Flipped Learning
    - Assessing how flipped learning approaches foster preservice teachers' digital skills and innovation in teaching practices is necessary.
    - Findings inform the design of technology-integrated teacher education programs and support the development of digital competencies.

To address the identified research gaps effectively, the following methodological approaches are recommended:

- Conducting longitudinal studies to assess the long-term effects of innovative teaching strategies on preservice teachers.
- Integrating virtual fieldwork within teacher preparation programs to identify the benefits, challenges, and best practices.
- Examining how mastery experiences influence teaching efficacy to identify effective strategies for enhancing preservice teachers' confidence and competence.
- Exploring the relationship between flipped learning and developing digital competencies to optimize technology integration in education.

Pursuing these research directions will enhance preservice teacher preparation, equipping future educators with the necessary skills and confidence to teach mathematical modeling effectively, thereby enriching their students' mathematical understanding and real-world application skills.

Conclusion

Summary of Key Findings

This article has systematically explored the transformation within mathematics education, focusing on the shift from traditional problem-solving methods to the incorporation of mathematical modeling. It has presented a detailed comparison and analysis of pedagogical strategies, emphasizing their impact on enhancing the teaching and learning of mathematical modeling for preservice teachers. The findings are organized into several key sections, each shedding light on distinct aspects of educational innovation in mathematics.

- *Traditional Problem Solving vs. Mathematical Modeling:* Mathematical modeling significantly diverges from traditional problem-solving approaches, and this has been meticulously explained in this article.
    - *Traditional Problem Solving:* Defined by its algorithmic and procedural emphasis, this method

is guided by George Pólya’s heuristics but critiqued for its lack of real-life applicability and abstract nature, which may hinder the development of critical thinking and adaptability.

- *Mathematical Modeling as a Pedagogical Shift marks* a significant evolution in pedagogy, prioritizing the application of mathematical concepts to real-world problems. This approach diverges from traditional methods by promoting a cyclic and interdisciplinary process, substantiated by research as superior for developing analytical and problem-solving skills.

- *Innovative Ways of Teaching Mathematical Modeling:* Several innovative ways of teaching mathematical modeling have been identified.
  - *Project-Based Learning (PjBL):* Recognized for its effectiveness in teaching mathematical modeling, PjBL engages preservice teachers in authentic projects, enhancing their mathematical problem-solving abilities and fostering a positive attitude towards STEM fields. This method aligns with constructivist and inquiry-based learning theories, emphasizing active engagement and exploration.
  - *Inquiry-Based Learning:* This approach significantly bolsters students' critical thinking and problem-solving capabilities by immersing them in exploration and experimentation. Inquiry-based learning improves science literacy and confidence, promoting a curiosity-driven learning experience.
  - *Technology-Enhanced Learning:* Highlights the role of digital tools in transforming the teaching of mathematical modeling, offering innovative ways to engage students and enhance learning outcomes. Integrating simulations, online platforms, and mathematical software enriches the educational experience.
  - *Flipped Classroom Approach:* This innovative strategy inverts traditional teaching methods to focus on active, student-centered learning. The flipped classroom model has increased student engagement and improved learning outcomes by utilizing class time for interactive learning experiences.
  - *Interdisciplinary Approaches:* Illustrate the benefits of integrating mathematical modeling with real-world applications from various disciplines. Such approaches enhance student engagement, facilitate the applicability of mathematics in STEM, and promote meaningful learning experiences.
  - *Collaborative Learning emphasizes* the social constructivist nature of learning, where students work together to solve problems and build knowledge. This method improves understanding of mathematical concepts and fosters positive attitudes towards mathematics, highlighting the importance of communication and collaboration skills.
  - *Realistic Mathematics Education (RME): This approach focuses on starting with 'real' contexts to enhance mathematical modeling abilities, using didactical models to facilitate students' understanding of mathematical concepts. It* prepares teachers for an inquiry-oriented approach and effective classroom behavior.
- *Designing Authentic Mathematical Modeling Tasks:* The study underscores the importance of incorporating real-world data and scenarios into mathematical modeling tasks. Making mathematical concepts tangible and relatable enhances students’ motivation and problem-solving skills.

- *Identified Challenges and Proposed Solutions:* Challenges such as mathematical anxiety, resistance to new teaching methods, and conceptual difficulties are addressed. Solutions include professional development, curriculum redesign, and promoting collaborative learning experiences, emphasizing the importance of interdisciplinary approaches and real-world applications.

The findings advocate for a pedagogical shift towards innovative teaching strategies that enrich mathematics education for preservice teachers. By integrating real-world applications, collaborative learning, and technology-enhanced teaching methods, educators significantly improve the teaching and learning of mathematical modeling, preparing preservice teachers to foster a deeper understanding and appreciation of mathematics in their future classrooms.

Limitations

This study on innovative approaches for teaching mathematical modeling to preservice teachers has several limitations that should be acknowledged. Firstly, the research primarily relies on qualitative synthesis, which may introduce subjectivity in interpreting findings from the literature. The inclusion criteria for selecting studies, though systematic, might have excluded relevant research due to publication bias or language limitations, as only English-language studies were considered.

Secondly, the study's focus on preservice teachers may not fully capture the complexities and challenges faced by in-service teachers or educators in different educational contexts. The specific needs and experiences of in-service teachers might require different pedagogical strategies and support mechanisms, which this study does not address comprehensively.

Thirdly, the review of innovative teaching methods, such as Project-Based Learning (PjBL), Inquiry-Based Learning (IBL), and technology-enhanced learning, while thorough, is limited by the variability in implementation across different educational settings. The effectiveness of these methods can vary significantly depending on factors such as institutional support, access to resources, and teacher preparedness, which are not uniformly accounted for in this study.

Moreover, the integration of real-world contexts into mathematical modeling tasks, though highlighted as beneficial, poses practical challenges that are not extensively explored in the study. These challenges include the availability of authentic data, the complexity of real-world problems, and the alignment of such tasks with curriculum standards and assessment practices.

Lastly, the proposed solutions for overcoming challenges in teaching mathematical modeling, such as professional development and curriculum redesign, require substantial institutional commitment and resources. The feasibility of these recommendations in different educational contexts, especially in under-resourced settings, is a critical consideration that the study does not fully address.

In conclusion, while this study provides valuable insights into innovative teaching strategies for mathematical modeling, future research should aim to address these limitations by incorporating more diverse educational contexts, employing mixed-method approaches, and exploring the practical implementation of proposed solutions in various settings.

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