Bridging the Gap: Understanding Students' Struggles with Algebraic and Graphical Representations of Functions

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Abstract

This study investigates the challenges students face in transitioning between algebraic and graphical representations of functions. Despite the equivalence of these forms, students show a strong preference for algebraic methods, with lower success rates on graphical tasks. Analysis of responses from 300 grade 12 STEM students in Nepal reveals a reliance on algebraic techniques and a lack of conceptual understanding of graphs. The study highlights the need for pedagogical adjustments to better integrate graphical reasoning into mathematics instruction. Recommendations include incorporating visualization tools and metacognitive training, along with curricula that balance algebraic and graphical methods to improve students' problem-solving flexibility.

Introduction

Functions are fundamental concepts in mathematics education, often considered the most important idea for understanding the subject (Dubinsky & Harel, 1992; Romberg, Fennema, & Carpenter, 2012). The introduction of algebraic and graphical representations marks a critical point in learning, as it helps students use one symbolic system to understand another (Leinhardt et al., 1990). The relationship between these two forms—graphs helping to analyze functions and algebraic expressions defining the same concepts—plays a key role in constructing mathematical understanding. However, research has shown that students often struggle to transfer their knowledge of functions across different contexts, particularly when moving between algebraic and graphical forms (Hitt, 1998; Knuth, 2000; Leinhardt et al., 1990). These challenges are evident not only in mathematics but also in other scientific disciplines, where understanding in one context does not guarantee proficiency in another (Dreyfus & Eisenberg, 1982; Even, 1990; Greeno, 1989).

Many students enter higher education with a weak grasp of functions and their graphs, often due to limited emphasis on graphical concepts in earlier education (Durant & Garofalo, 1994). Even those who study functions in high school often struggle with them (Markovits et al., 1988; Leinhardt et al., 1990). Graphical literacy is essential for both mathematics and science education.

Shuard and Neill (1977) argue that all students should be able to interpret graphs, emphasizing that those who cannot are disadvantaged. However, different perspectives in mathematics and science about the role of graphical literacy can create teaching inefficiencies (Booth, 1981). Graphs serve multiple purposes in mathematics, from recording data to representing relationships and functions, with predictive value being a critical aspect (Shuard & Neill, 1977).

This article presents findings from a study that assesses students' ability to transfer skills between algebraic and graphical representations of functions. It explores why students struggle to apply algebraic knowledge to graphs and vice versa. The research was inspired by years of teaching high school and college algebra, where consistent difficulties in bridging algebraic and graphical representations were observed. This pattern, evident across multiple semesters and countries, raised the question of whether it reflects a broader trend.

Method

The study's design and questionnaire were inspired by the work of researchers such as Eric J. Knuth, Van Dyke and White, and Fernando Hitt in the field of functions and their graphical representations (Knuth, 2000; Van Dyke & White, 2004; Hitt, 1998). To assess students' ability to connect different representations of functions, a random sampling technique was employed. The sample consisted of 300 grade 12 STEM students from various high schools in Kathmandu, Nepal, who had studied graphing in grade 11 and its applications in calculus. These students were expected to demonstrate proficiency in both algebraic and graphical methods.

Participants completed a questionnaire with five problems designed to evaluate their ability to interpret functions both algebraically and graphically. They were given 45 minutes to solve the problems and encouraged to explain their reasoning to assess their visualization and transformational skills. The problems involved linear, quadratic, square root, and rational functions, requiring students to connect algebraic solutions with graphical concepts such as intercepts, solution points, domain, and range. For example, one question asked students to find x and y values either algebraically or graphically and then identify them as the intercepts of a line.

While algebraic and graphical representations of functions convey the same information, they differ in computational nature (Larkin & Simon, 1987). Graphs explicitly display solutions, domain, and range, while algebraic expressions require computation to extract this information. Despite these differences, both representations are informationally equivalent—they can each be derived from the other. This study hypothesizes that students who recognize the connection between the two forms will opt for more efficient problem-solving strategies, often preferring graphical methods.

Data Analysis

Responses were analyzed using a cognitive framework that integrates both constructivism and metacognition. Constructivism (Steffe & Gale, 1995) was applied to examine how students build on their prior knowledge of algebra and graphing, while a metacognitive lens (Hacker, Dunlosky, & Graesser, 2009) was used to explore students' awareness of different methods and their ability to adapt to the problem at hand. Together, these frameworks offer a comprehensive understanding of students' problem-solving strategies. This combined insight informs instructional approaches that aim to balance symbolic and visual reasoning, helping students develop more flexible and effective problem-solving skills.

Results

Question 1

The graph of the equation $2x + 3y - 6 = 0$ is given below.

(a) What value of x gives $y = 2$?

(b) What value of y gives $x = 3$?

(c) Can you answer the questions in part (a) and (b) by a method different than the one you used? Explain your answer.

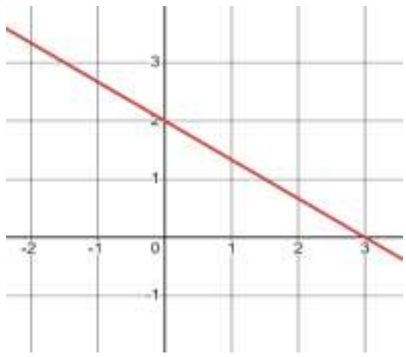

Figure 1. Graph of a Linear Function

The problem assesses whether students can interpret a linear equation both symbolically (through algebraic computation) and visually (by analyzing the graph). Students must realize that the same problem can be solved using either an algebraic approach or a graphical method. The third part of the question explicitly asks students to reflect on alternative methods.

Out of 300 students, 290 participated students responded at least one part of the question. Of these, 195 students (about 67% of the respondents) chose to answer parts (a) and (b) algebraically, showing a clear preference for algebraic methods, even though the graphical method is often quicker and easier. Only 95 students which is nearly 33% of the 290 students chose to answer it graphically. This preference for algebra over graphical solutions was consistent across both parts. In part (c), only 178 students responded to the question. Out of 195 who answered part (a) and (b) algebraically, only 83 responded to the question in part (c), and of these 83, only 60 students were aware of the alternative approach for part (a) and (b). Hence 155 (about 52%) students were aware of graphical and algebraic approach of solving the problem whereas 145 (about 48%) students either had no idea at all or no graphical approach of solving the problem. Some students expressed this lack of awareness of the graphical approach with statements like:

- “No, I cannot answer the question by a different method.”
- “There is no other method to answer the question in part (a) and (b).”
- “Sorry, I have no clue.”
- “It can’t be explained, sorry.”

This indicates that while students overwhelmingly began with algebraic methods, many of them only realised the possibility of a graphical solution when confronted with part (c). As one student noted, “The easiest way to do it is by putting $x = 0\ y = 0$," and highlighting the strong reliance on algebraic techniques. The responses in part (c), particularly the statements like “I have no clue” and “There is no other method to answer the question,” suggest a lack of metacognitive awareness. Many students were not consciously aware that graphical solutions were an option, indicating that they were unaware of alternative methods beyond the algebraic ones they were comfortable with. The graph below summarizes the outcomes of question 1.

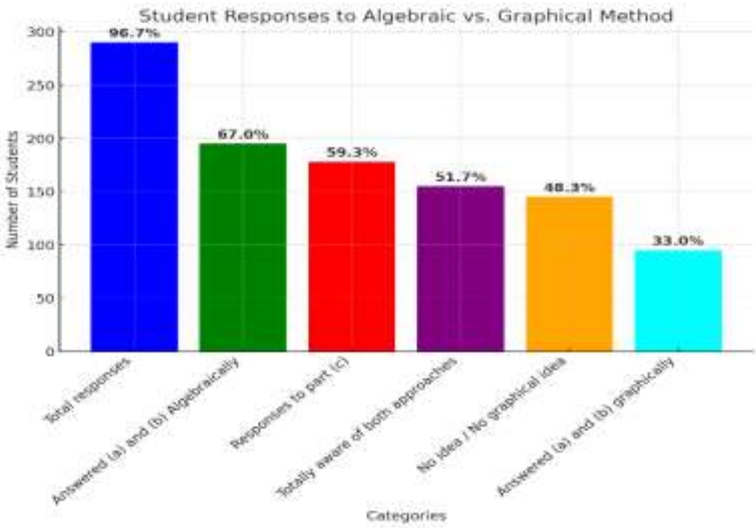

Figure 2. Students’ Response to Question 1

Question 2

The graph of a linear function $?x + ?y = 6$ is given below.

(a) Can you find some obvious solution of the given equation without the missing coefficient? Explain your answer.

(b) If possible, find the missing coefficient of the equation. Explain your answer.

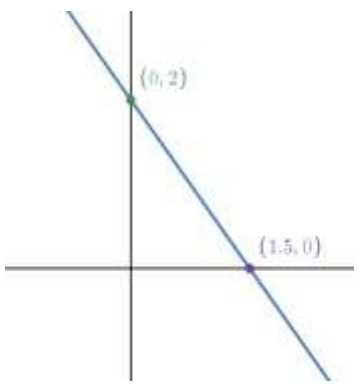

Figure 3. Graph of a Linear Equation

The purpose of this problem was to assess whether students can relate the graph to the equation (e.g., recognizing intercepts as direct solutions), whether they express a preference for graphical or algebraic methods and justify their choice, and if they demonstrate metacognitive awareness by identifying mistakes and adjusting their approach accordingly.

Out of 300 students, only 44 (14.67%) answered correctly, while 51 (17%) provided partially correct answer of part (a)—95 students (about 32%) gave either correct or partially correct answer. Of these 95 students, only 20 students (6.6%) demonstrated a clear understanding of the connection between the equation and its graph. The majority—205 students (68%)—showed little understanding of the question or its graphical representation. Many students were confused by the missing coefficients of x and y in the equation. Some identified the midpoint of the given line as the solution, while others suggested that a complete equation would allow them to check the points on the graph by substitution, though they failed to recognize that these points are solutions to the equation while in part (b) which requires an algebraic approach—use of slope-point formula of a straight line—to find the missing coefficients about 55% students—significantly higher than in part (a)—students answered either correctly or partially correctly—reflecting their strong reliance on algebraic skill of solving a problem.

Many students believed that finding the "obvious" solutions required solving for the coefficients, with one student commenting, “Without solving, we can’t find the missing coefficients.” This reliance on algebra is further highlighted by a student who wrote, “We can’t find the obvious solution without the missing coefficients, as they determine the constant nature of the graph.” Interestingly, a few students first found the equation of the line using the two-point formula and then verified the given points on the graph, demonstrating their heavy reliance on algebraic skills.

Students struggled to connect algebraic equations with their graphs. While they were more comfortable with algebraic methods due to practice, their ability to interpret graphs was less developed. Their metacognitive awareness of algebra was stronger, but they lacked reflection when using graphs, leading to confusion. More metacognitive training is needed to help students consider alternative problem-solving methods and improve their understanding of graphs. The graph below gives a visual outlook of the results of question 2.

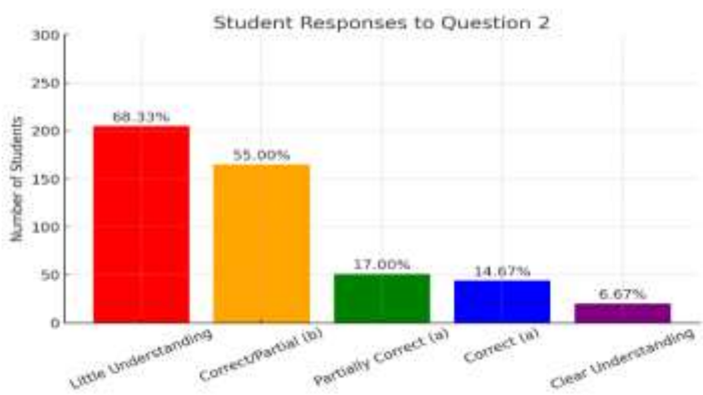

Figure 4. Students Response to Question 2

Question 3

Refer to the graph below to answer the questions.

a) $f(4) = g(-2)$

b) $f(4) = g(4)$

c) $f(-2) = g(-2)$

d) More information is necessary to say something definitive about $f(x)$ and g(x).

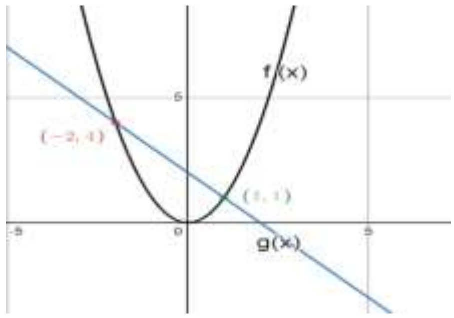

Figure 5. Graph of a Straight line and Parabola

The question requires students to interpret the graph of two functions, *f(x)* and *g(x)*, and make connections between their values at specific points. According to the *constructivist* framework, students will build their understanding through interaction with the graph, recognizing how the two functions relate to one another. The focus would be on their ability to construct meaning from the graph and draw conclusions based on the points of intersection.

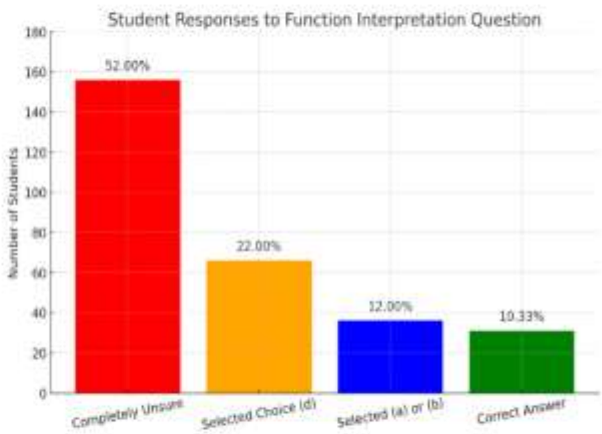

Figure 6. Students' Responses to Question 3

Out of the 300 students, only 31 (about 10%) answered the question correctly. A surprising 52% of students were completely unsure of what the question was asking, with one student commenting, "I don't know. Very bad question." Additionally, 66 students (22%) selected choice (d), believing that there was insufficient information to answer the question. Around 12% chose answers (a) or (b). In total, 90% of students either selected an incorrect answer, gave an unclear response, or did not attempt the question at all. This outcome is particularly surprising, given that many students struggled to understand the domain and solution points of a function, even when a graphical representation was provided. Some students even attempted to find the algebraic form of the function from the graph to determine the correct answer. Despite the relatively low level of difficulty of the problem, the percentage of correct responses was alarmingly low. This result mirrors the trends observed in previous problems, where students showed a strong reluctance to use graphical representations and instead relied heavily on algebraic methods. Bar graph below gives an outlook of the results of question 3.

Question 4

The graph of the function $f(x) = \sqrt{(x-1)}$ is given alongside. Answer the following questions.

a) Find the domain of f. Demonstrate or explain how you found it.
b) Find $f(0), f(1), f(5), f(6)$.
c) Find the x values on the domain of f that correspond to the points A, B, C, D. Includes details in your answer.

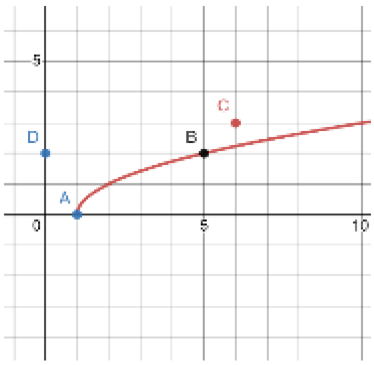

Figure 7. Graph of a Square Root Function

This task's problem analysis framework examines how students approach problem-solving, focusing on identifying key strategies and evaluating various methods. Students must apply their knowledge of domains, evaluate function values, and understand the connection between algebraic expressions and their graphs—essential concepts in high school mathematics (National Council of Teachers of Mathematics [NCTM], 2000; Common Core State Standards Initiative [CCSSI], 2010). These concepts are introduced in grade 11 in Nepal and must be carried forward to grade 12.

Based on students' responses in part (a), only 42 out of 300 students (14%) understood the domain and attempted the problem graphically. In contrast, 79 students (26%) understood the concept but approached the problem algebraically. However, 179 students (60%) had no clear understanding of how to approach the problem. This highlights a significant gap in students' understanding of fundamental concepts like domain, even when different function representations are provided.

The aim of this question in part (b) was to assess whether students recognize that $x = 0$ is excluded from the domain found in part (a) and to evaluate their willingness to use the graph to find values at various points within the domain. The survey results show that only 22 students (about 7.5%) correctly identified that $f(0)$ is undefined, as $x = 0$ is not part of the domain. Meanwhile, three out of four students attempted the problem without understanding this concept, simply plugging in x-values into the equation. Many of these students recognized that $f(0)$ resulted in an imaginary number and responded with terms like "undefined," "impossible," or "infinity." However, there was insufficient evidence to suggest they understood that $x = 0$ is not part of the domain, and thus $f(0)$ cannot be computed either algebraically or graphically. In contrast, more than half of the students—189 out of 300—had no idea how to approach the problem. They either did not attempt it or provided incorrect answers.

As seen in part (b), a similar pattern emerged in part (c) too. Only 16 students (about 5.5%) met the expectations of the problem and answered correctly. In contrast, 103 students (approximately 34%) answered the question without realizing that points C and D cannot correspond to any x-values, as they are not solution points of the equation. One student wrote, "By studying the graph, I found the values of x for points A, B, C, and D," despite not understanding that C and D are not on the graph. Meanwhile, 63% of students either had no idea how to approach the problem or gave incorrect answers. Below is an example of a student's response.

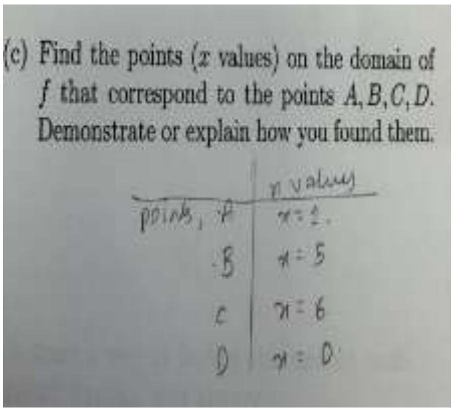

Figure 8. A Response of a Student

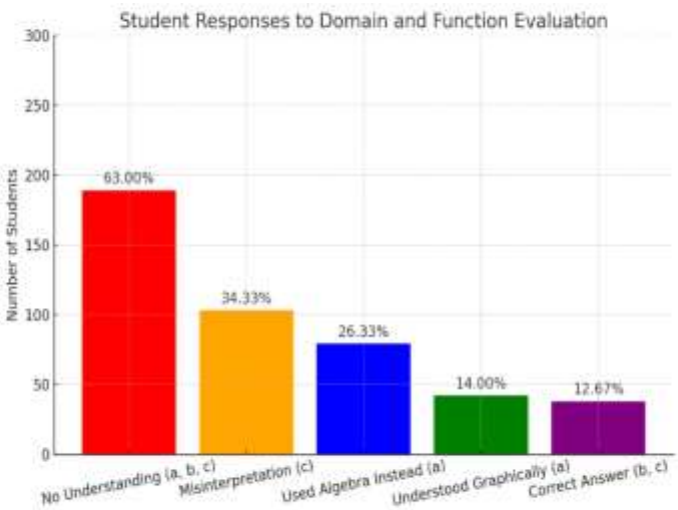

Figure 9. Students' Responses to Question 4

In conclusion, there is a significant gap in students' understanding of the domain of the function and how to evaluate points on the graph. At the same time, a large portion of students failed to recognize key domain restrictions and the inability of certain points (C and D) to correspond to any x-values. Also, many students struggled to use the graph effectively, and there was widespread confusion around function evaluation and domain concepts. The bar graph below gives a visual perspective of the outcome of the question 4.

Question 5

Answer the following questions. Try to include as much detail as possible in your answer.

a) Find the domain of the function $f(x) = \frac{x-2}{x+2}$.

b) Find the domain of the function whose graph is given below.

c) Which question (a) or (b) did you find easier to answer? Explain.

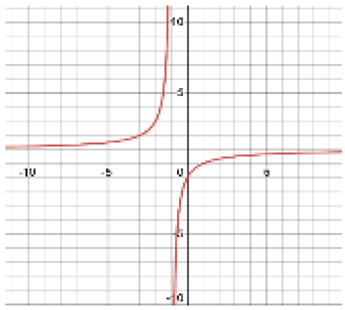

Figure 10. Graph of a Rational Function

The objective of this task is to assess students' understanding of domains of functions, including restrictions caused by undefined values. The cognitive demand requires algebraic reasoning to determine where $f(x)$ is undefined, interpretating a graphical representation to determine domain restriction, and evaluates students' perceived difficulty in switching between algebraic and graphical methods.

The outcome of the survey indicates, only 75 students (25%) correctly identified the domain in part (a), and an unexpectedly low number—only 25 students (about 9%)—were able to identify the correct domain in part (b) where a graphical representation of the function was provided. This low number is striking, especially given that 275 students struggled to approach the problem in part (b). In part (c), 27% of students felt comfortable finding the domain when presented with the algebraic form, while only about 10% found the graphical method easier. Surprisingly, 62% of respondents did not select any method as a compatible approach for finding the domain, and only 2% felt equally comfortable with both methods.

These results suggest that the majority of students lack a clear understanding of the domain of a function, regardless of how the function is represented. A clear pattern emerged showing that more students were comfortable determining the domain when given the algebraic representation. Many students noted that part (a) was easier because the equation was directly provided. Here are a few student comments from part (c) that reflect this preference:

- "(a) is a bit easier than (b) because the direct function is given in this question. But in (b), the function should first be determined."
- "(a) is easier to answer since the function is straightforward, but (b) is more difficult since the function in the graph is not easy to identify."
- "(a) was easier because the graph is difficult to understand."

From these comments, it's clear that many students find it easier to determine the domain when the function is given algebraically. They expressed difficulty in part (b) because they first needed to determine the function from the graph before identifying its domain. Additionally, some students commented on how the education system in Nepal places greater emphasis on algebraic and computational problem-solving methods, leaving students less comfortable with graphical approaches:

- "I found (a) easier because we have done this type of problem before using just one method. We are not as familiar with the graphical approach since our education system focuses more on algebraic methods."
- "(a) is easier because solving algebraically is easier, and we're not used to graphical assignments."
- "I found (a) easier because it was taught more in class, and the teacher focused more on part (a) than part (b)."

Students showed a strong preference for algebraic methods over graphical ones when determining the domain of a function. While 25% correctly answered part (a) algebraically, only 9% succeeded with the graphical approach in part (b). Many struggled with graphs due to limited exposure, as their education emphasised algebraic computation. This highlights the need for greater focus on graphical reasoning to improve students' ability to interpret functions across different representations. Bar graph below is offers a visual representation of the outcome of the question 5.

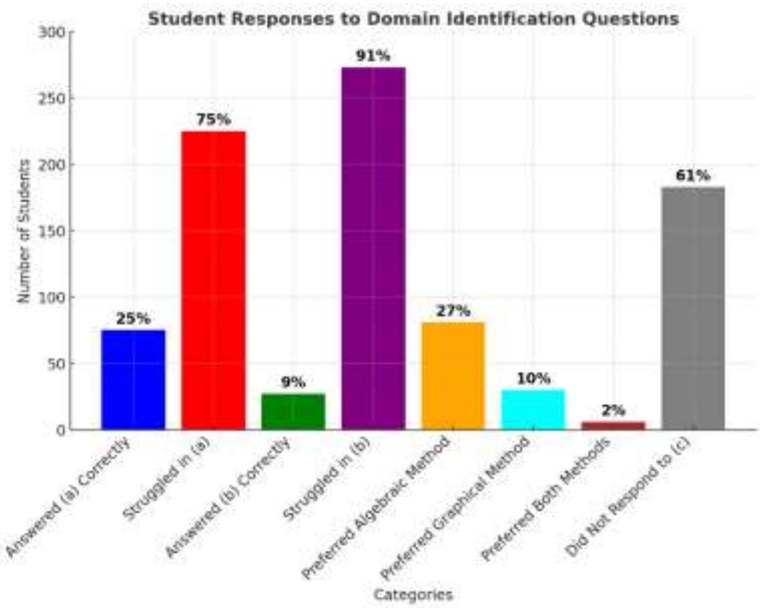

Figure 11. Students' Responses to Question 5

Discussion and Conslusion

The analysis of student responses reveals a strong reliance on algebraic methods over graphical representations, limiting their problem-solving flexibility. Across multiple questions, students overwhelmingly preferred symbolic manipulation, even when graphical approaches provided more intuitive solutions. In Question 1, 67% of students solved problems algebraically, while only 33% attempted a graphical approach. Similarly, in Question 5, 25% of students correctly determined the domain algebraically, whereas only 9% succeeded using the graphical representation. This pattern indicates that students find algebraic techniques more accessible and familiar, likely due to their prior mathematical training. Furthermore, many students demonstrated limited awareness of alternative solution methods. In Question 1, nearly half of the students either had no idea about alternative approaches or explicitly stated that no other method existed, reflecting a rigid approach to problem-solving.

Many mathematics educators may consider these questions routine exercises that student are expected to understand. In Nepal, students are introduced to these concepts in grade 11, and the survey was conducted at the start of the 12th-grade academic year. Despite the few-month gap between when students learned these concepts and when the survey took place, the concepts are foundational for 12th grade. An interesting observation is how well students retain previously learned concepts. Teachers typically provide little or no review during the transition from grade 11 to grade 12, despite the importance of these concepts. As Schoenfeld, Smith, and Arcavi (1993) highlight, the retention of mathematical ideas can be hindered when there is little effort to review or revisit the material during this period. This may explain why students struggled with questions requiring both algebraic and graphical reasoning, as their ability to recall and apply previously learned concepts appeared to be inconsistent.

In addition, the results highlight widespread difficulties in interpreting graphs and understanding function properties, particularly domain restrictions and function evaluation. In Question 4, 60% of students failed to grasp the concept of domain even when provided with a graph, while in Question 2, 68% struggled to connect algebraic equations to their corresponding graphical representations. In Question 3, only 10% of students answered correctly, while 52% were entirely unsure of what was being asked, with some commenting that the question was unclear or difficult. This outcome closely aligns with findings from a U.S.-based study (Van Dyke & White, 2004), where students also demonstrated significant difficulties in extracting information from graphs and making meaningful interpretations. However, the students who participated in this survey performed noticeably worse than their American counterparts in that study, where a higher percentage of students were able to answer correctly or at least attempt a meaningful interpretation. This suggests that the difficulty in using graphical representations is not only a general trend across different educational contexts but may also be more pronounced in this particular group of students, potentially due to gaps in retention and review of key mathematical concepts.

The findings show that students primarily favor algebraic methods over graphical reasoning, even when graphical approaches may be more intuitive and efficient. Research indicates that students who prefer visual problem-solving tend to perform better in tasks involving graphical representations, although this preference does not correlate with their performance in algebraic contexts (Tiew et al., 2023; Mainali, 2021; Orhun, 2012). These results align with similar studies in the United States (Knuth, 2000a; Knuth, 2000b; Van Dyke & White, 2004), where students also favored algebraic approaches over graphical ones. However, a significant difference is that students in this study performed notably worse on graphical problems, with only 10% correctly answering the *question 3* of this study that is similar to one in Van Dyke and White's (2004) study, where 32% succeeded. This trend persisted across other graphical problems.

A curriculum that equally values both algebraic and graphical methods could foster a more flexible, deeper understanding of mathematical concepts (Hiebert & Carpenter, 1992; Booth & Newton, 2012; Fey & Harel, 2004). The results of this study suggest that the current curriculum, textbooks, and teaching practices may place too much emphasis on symbolic manipulation, overlooking the vital role of visual representation in problem-solving. An analysis of high school textbooks in Nepal (grades 9-12) shows that 84.4% of problems (3,017 questions from selected exercises) focus on procedural skills, with nearly 95% not requiring graphical responses (S. Pathak, E. Paudel, 2024). This underscores the need for a curriculum that integrates graph-based problem-solving more effectively and emphasizes the conceptual connections between equations and their graphical representations. Additionally, clear guidelines for textbook design are necessary to ensure a more balanced approach.

Duval (2006) emphasizes that multiple representations are crucial for mathematical understanding, and students often struggle to transition between algebraic and graphical forms—a pattern seen in this study. This challenge is evident in the significant difficulty many students have with basic graphical concepts, such as identifying solutions or determining domains visually. To address this, instructional strategies should focus on enhancing students' ability to interpret and analyze graphs. Tools like graphing software and dynamic visualizations could help bridge this gap by providing interactive, visual learning experiences.

Additionally, the study reveals that many students have trouble recalling previously learned material, particularly concepts introduced in earlier grades. This suggests a need for more consistent reinforcement of fundamental topics through spaced repetition and cumulative assessments (Rohrer & Pashler, 2007; Bjork & Bjork, 2011; Brown et al., 2014). Regularly revisiting past concepts would improve retention and knowledge application. Students' preference for algebraic methods appears influenced by how they were taught, with a strong focus on symbolic problem-solving and limited exposure to graphical reasoning. This reflects a broader issue in education, where procedural skills are prioritized over conceptual understanding.

Given the challenges students face with graphical reasoning, educators and curriculum designers should reconsider how mathematical understanding is assessed. Traditional assessments tend to focus on algebraic manipulation, reinforcing students' reliance on this approach. By incorporating more graph-based and conceptual questions into exams and assignments, educators can promote a more holistic understanding of mathematical relationships (NCTM, 2000; Artigue, 2009; Schoenfeld, 1992).

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